ESSAY

AGGREGATION OF PROBABILITIES AND ILLOGIC

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[*] Ziff Professor of Law, Cornell University. This Essay's argument draws on a part of Kevin M. Clermont, *Death of Paradox: The Killer Logic Beneath the Standards of Proof*, 88 NOTRE DAME L. REV. (forthcoming 2012), *available at* http://ssrn.com/abstract=1986346, which more fully documents the argument and broadens its reach considerably.

I. INTRODUCTION

Scholars often invoke a paradox involving multiple independent theories that are alternative routes to the same conclusion, whether that conclusion is a judgment in a case with multiple counts or a decision on a particular claim or element of a claim. These observers lament that the law denies relief to a supposedly deserving plaintiff (or to a defendant with multiple defenses almost proved):

> Consider a case involving three different legal theories and three different factual foundations. Plaintiffs deserve to win if one of the stories embodying one legal theory is true; defendants deserve to win only if all of their competing stories are true (for if this is false, one of the plaintiff's stories is true). For example, assume the plaintiff has alleged defective design, defective manufacture, and failure to warn theories. If the probability of each is .25, the "probability" of each not being true is .75, but, the probability of at least one being true is $1 - .75^3 = .58$, and perhaps plaintiff should win, even though the individual probabilities of each being false is .75.[1]

The lamenters are quite serious, sometimes abandoning the restriction that theories be alternative routes to the same conclusion and even extending their point to parallel conclusions in the criminal law. Thus, some startlingly argue for *conviction* on the basis of a number of offenses *almost* proved.[2]

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[1] Ronald J. Allen & Sarah A. Jehl, *Burdens of Persuasion in Civil Cases: Algorithms v. Explanations*, 2003 MICH. ST. L. REV. 893, 939.

[2] For example, in their article *Aggregating Probabilities Across Cases: Criminal Responsibility for Unspecified Offenses*, Professors Alon Harel and Ariel Porat write:

> Should a court convict a defendant for an unspecified offense if there is no reasonable doubt that he committed an offense, even though the prosecution cannot prove his guilt as to a particular offense beyond a reasonable doubt? Stated otherwise, is committing *an* offense sufficient for a conviction or must a prosecutor establish what this offense is to justify a conviction? [We] contend[ ] that, under certain conditions, a prosecutor should not have to establish the particular offense committed by a

Others have tried to explain, and thereby to justify as a practical or prudential matter, why the law illogically ignores the paradox.[3] In a recent article in the *Yale Law Journal*, Ariel Porat and Eric A. Posner go on the offense.[4] They attack the law's general refusal to multiply the probabilities of claims and defenses, and then argue for overhauling the law to cure this logical mistake.[5] Their *Aggregation and Law* is an article beautifully developed by admirable scholars. But it rests squarely on the premise that refusal to multiply probabilities in legal fact-finding is illogical.

Is it? Few indeed have questioned whether ignoring the paradox is illogical. I do. My position is that the paradox that motivated Porat and Posner's article simply does not exist. Instead, modern logic demonstrates that the tort plaintiff who supports three theories each to .25 has proved his case to only .25 and so should lose—just as the plaintiff loses under current law. Therefore, the law should not, and does not, aggregate by multiplying probabilities.

> defendant—proof that the defendant committed an offense should be sufficient.

94 MINN. L. REV. 261, 261–62 (2009). On aggregation in criminal sentencing, see Kevin Bennardo, *A Quantity-Driven Solution to Aggregate Grouping Under the U.S. Sentencing Guidelines Manual*, http://ssrn.com/abstract=2148133 (Sept. 20, 2012).

[3] *See, e.g.*, Frederick Schauer & Richard Zeckhauser, *On the Degree of Confidence for Adverse Decisions*, 25 J. LEGAL STUD. 27, 41–47 (1996) (conceding the probability argument regarding multiple crimes almost proved, but arguing that the criminal law still should not convict for reasons of abundant caution).

[4] Ariel Porat & Eric A. Posner, *Aggregation and Law*, 122 YALE L.J. 2 (2012). Astoundingly, Professors Porat and Posner double down on the criminal law example. Not only would they convict for one crime out of two separate crimes *almost* proved, but they would acquit for one crime out of two even when each is proved beyond a reasonable doubt but the odds of the defendant's having committed *both* crimes is mathematically less certain (after all, they argue, 96% x 96% = 92%). *Id.* at 35–36. I refute this fallacy in Kevin M. Clermont, *Conjunction of Evidence and Fuzzy Logic* 16–21 (Cornell Legal Studies Research Paper No. 12–58, 2012), *available at* http://ssrn.com/abstract=2115200.

[5] Porat & Posner, *supra* note 4, at 9.

II. LOGIC SYSTEMS

A. LOGICAL ASSUMPTIONS

We tend to forget that our classical logic system rests on the critical assumption of bivalence: every proposition must either be true or be false, taking a value of 1 or 0. The traditional probability theory built on classical logic rests on the same assumption: probability represents the random uncertainty that the proposition is actually true or false, one or the other.[6] The bivalent assumption pays off, in that classical logic turns out to be very useful. But like Euclidean geometry and Newtonian physics, classical logic is a very useful oversimplification that gives wrong answers around the edges of its assumptions.[7] Surprisingly, the law's task of fact-finding is on the edge. Classical logic starts delivering wrong answers when the fact-finder needs to conjoin elements or aggregate theories.

The cure lies in deploying a more general logic system. Many-valued logic, of which fuzzy logic is the most familiar variant, does not assume bivalence. It allows a proposition to be true to a degree, taking some value between 1 and 0.[8] It uses a degree of membership in a *fuzzy* set running from 1 to 0, in place of *crisp*-set membership classified as yes/no or as either 1 or 0.[9]

Take as an example the set *A* of men from five to seven feet tall. This would be a crisp set. Contrast the set *H* of men somewhere near six feet. It is a fuzzy set. So, not-so-tall-at-all Tom might be completely in set *A* but have a degree of membership in *H* of .25. If a classical probabilist says, "There is a 25% chance that Tom is tall," the speaker supposes that Tom is either tall or not tall, and given imperfect evidence he thinks that it is only 25% likely that

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[6] Clermont, *supra* note 4, at 3.

[7] *See* THEODORE SIDER, LOGIC FOR PHILOSOPHY 72 (2010) ("There are many reasons to get interested in nonclassical logic, but one exciting one is the belief that classical logic is *wrong*—that it provides an inadequate model of (genuine) logical truth and logical consequence."). By the way, Euclidean geometry and Newtonian physics, both based on classical logic, break down in the face of general relativity and quantum mechanics, which take down classical logic's bivalence assumption. *See* Michael Dummett, *Is Logic Empirical?*, *in* TRUTH AND OTHER ENIGMAS 269 (1978).

[8] *See* Clermont, *supra* note 4, at 4–7 (summarizing the basics of fuzzy logic).

[9] *Id.* at 6.

Tom would end up in the tall category upon accurate measurement. But when a fuzzy logician says, "Tom's degree of membership within the set of tall men is .25," he means that Tom is not very tall at all. The difference is real and considerable. It derives from the fact that the classical probabilist is assuming bivalence, so that one is tall or not, while the fuzzy logician is speaking of a world where one can be more or less tall. As the evidence improves, the chance of not-so-tall-at-all Tom's being tall will drop toward zero, but the fuzzy measure will hold steady. If someone is interested in Tom's tallness, the fuzzy measure is both more natural to seek and more useful to know.

Fuzzy logic can treat much more than this kind of vagueness. It can handle all kinds of facts subject to proof. Fuzzy logic can treat "event imprecision," or what some may call "normative uncertainty": was what Tom did blameworthy or, more accurately, to what degree was it blameworthy? But it can also handle "event occurrence," or what some may call "factual uncertainty": how likely is it that Tom did those acts?

Indeed, it can encompass all of traditional probability, by expressing probability as membership in a set—that is, probability is the degree to which the imagined universe of all trials would belong to the set of successful trials. A resultant virtue is that fuzzy logic can combine measures of event occurrence and event imprecision. It provides the common currency for measuring a 70% chance of occurrence of a .70 degree of fault.

We can thus envisage the fuzzy set of true facts and ask, for a particular proposition, what its degree of membership in that set is. Such degrees of truth turn out to be an excellent way to capture the kinds of uncertainty that most bedevil law.

In sum, degrees of truth constitute an inclusive measure of uncertainty, built right into the fuzzy logic system. This system therefore does not need to append bivalent logic's probability theory for random uncertainty, which incidentally is an add-on that can accurately handle only one of the various kinds of uncertainty that the world throws at us. Still, fuzzy logic ends up looking much like bivalence plus probability because its multivalent logic is the more general system. It includes bivalent logic as a special case. That is, a world of black or white is a special, extreme case of the world actually shaded in grays.

Let me interject here that the choice of logic system, classical or fuzzy, does not pivot on "how one views the world." It turns on how one chooses to represent the world. The advantage of using the more inclusive and less simplified logic system of multivalence is that it gives more accurate answers in more situations.

B. LOGICAL OPERATORS

With assumptions made, any logic system attains full definition by stipulating a small but adequate number of logical operators, such as intersection (or conjunction), union (or disjunction, aggregation, or OR), and negation.[10]

The operators of classical and fuzzy logic are quite similar. For aggregation of statements x and y, both use the MAX rule:

$$\text{truth}(x \text{ OR } y) = \text{maximum}(\text{truth}(x), \text{truth}(y)).$$

In classical logic, given the assumption of bivalence, the MAX rule yields truth(x OR y) = 1 if either x or y is true, but 0 otherwise.

In both classical logic with a probability overlay and fuzzy logic, the MAX rule will reduce to De Morgan's rule for aggregation—*if* one assumes bivalence and then adds an assumption of independent statements. De Morgan's rule provides that the aggregation of two independent statements (say, each .25 likely) equals the negation of the product of the negations of those statements ($1 - .75^2 = .44$).[11] For interdependent statements A and B, the product-rule portion of De Morgan's rule would involve P(A) and P(B|A), where the latter term is a conditional probability to be read as the probability that B will occur if A is known certainly to have occurred.

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[10] *See* SIDER, *supra* note 7, at 69 (stating that "a set of connectives is *adequate* [if, but only if,] all truth functions can be symbolized using sentences containing no connectives not in that set" and noting that a set of connectives that includes intersection, union, and negation is adequate for two-valued logic).

[11] *See* IRVING M. COPI, CARL COHEN & KENNETH MCMAHON, INTRODUCTION TO LOGIC ch. 14 (14th ed. 2010).

The explanation of De Morgan's rule is that if values can be only 1 or 0, then the probability measures of, say, 50% and 40% for two propositions mean no more than an expectation that in five out of ten and four out of ten times the value will turn up as 1. Thus, given the assumption of independence, only in half of the four times when the second proposition is 1 will the first be 1—that is, 20% of the time. As to aggregation, 70% of the time one or the other proposition, or both, will be 1. The MAX rule then tells us that in those 70% of the cases, truth(x OR y) = 1.

De Morgan's rule applies in many situations, namely, those dealing with randomly uncertain estimates of independent events' likelihoods in binary circumstances. Think of drawing black or white balls from urns. But are Professors Porat and Posner correct in thinking that it should apply to legal fact-finding?

The alternative approach would be not to assume bivalence. Let H be one fuzzy set and B be another fuzzy set in the universe. The two sets might be independent, in the sense that the degree of membership in one set has no effect on the degree of membership in the other set, but they need not be. Designate the membership of element x in H as truth(x), and the membership of element y in B as truth(y). Now, fuzzy logic says to stick with the more general MAX rule. Under it, the truth of the aggregation equals the larger of truth(x) and truth(y).[12]

For a more specific example, let H be the set of tall men and B be the assumedly independent set of smart men. So, if Tom is a .25 member of H and a .40 member of B, then Tom is a .40 member of the set of either tall or smart men. The aggregated set becomes larger, so Tom's degree of membership in it does not increase above the greater of his tallness and smartness levels. Significantly, we are not interested in the odds of Tom being completely tall or completely smart, the odd question that De Morgan's rule would answer.

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[12] In other words, the union of sets H and B is a third fuzzy set, C, where membership of element z in C is defined as truth(z) = maximum (truth(x), truth(y)). Michael Laviolette et al., *A Probabilistic and Statistical View of Fuzzy Methods*, 37 TECHNOMETRICS 249, 251 (1995).

Fuzzy logic would apply the same MAX rule if one were trying to determine if Tom either committed a tort or breached a contract. The MAX rule would displace De Morgan's rule.

Ay, there's the rub: the MAX rule gives a different answer than De Morgan's rule gives for aggregating theories. Which rule does the law apply? The law applies the MAX rule. Why? Because De Morgan's rule applies only if classical logic's assumption of bivalence holds for fact-finding, and it does not.

III. MULTIPLE THEORIES

Let me use the aggregation, or multiple-theory, paradox to explain that nonintuitive point about bivalence. Imagine a claim A that is 25% likely to be valid and an independent claim B that is 40% likely to be valid.

Some scholars, but not the law, say that the likelihood of (A OR B) is equal to $1 - (.75 \times .60) = 55\%$, and therefore the plaintiff should win.[13] I am saying, however, that the likelihood of (A OR B) is equal to that of the likelier of A and B, that is, 40%, and therefore the plaintiff should lose.

It is easy to say that I am being obtuse, because anyone can intuit that the probability of A and B's aggregation must be higher than either. But the fact that the law says otherwise, that a widely accepted logic system built on multivalence says otherwise, and that some pretty bright lights shining in philosophy's vast literature on vagueness say otherwise[14] should give pause to that intuition. Let me present three hypotheticals to suggest my counterargument.

First, for modeling event imprecision, imagine a claim A for tort, where the defendant did acts that were 25% bad, and an independent claim B for contract, where the defendant did acts that constituted 40% of what would be an unquestionable breach. These are members of fuzzy sets. The breaching quality of the acts

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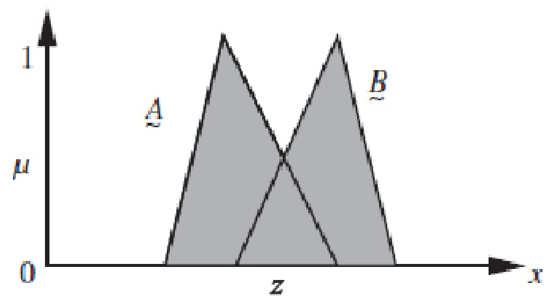
[13] *See, e.g.*, Porat & Posner, *supra* note 2, at 4–5.

[14] *See, e.g.*, BERTRAND RUSSELL, HUMAN KNOWLEDGE: ITS SCOPE AND LIMITS 359–61 (1948) (arguing that his "degrees of credibility" do not follow the product rules of traditional probability). A fundamental problem here is that legal academics debate such matters while ignoring this literature.

has no bearing on the tortiousness of the acts, that is, the 40% showing has no effect on the 25% showing. The likelihood of their aggregation is 40%, so that we can say only that the plaintiff went 40% of the way toward proving liability. Here is a way to visualize the aggregation of two fuzzy sets *A* (acts constituting a tort) and *B* (acts constituting a breach):[15]



The degree (μ) to which a set of facts (z), located on the x-axis, constitutes either a tort or a breach is indicated by the membership line for *A* or *B*, whichever is higher at point z. Another way to understand aggregation is to realize that this hypothetical is just like not-so-tall-at-all and not-so-smart Tom. A fact-finder would have no interest in the chance that Tom is either completely tall or completely smart; in any event, De Morgan's rule would yield an aggregation of about 0% likelihood, because the chance of Tom's being completely tall and the chance of his being completely smart are about 0% and 0%. Instead, the fact-finder would be interested in Tom's degrees of membership in the set of tall men (25%) and in the set of smart men (40%); he is therefore a .40 member of the set of tall or smart men.

Second, for modeling bivalent chance, imagine a ball A drawn from an urn with only 25% black balls among white balls, and a ball B drawn from another urn with 40% black balls. The chance that at least one of them will be black (and let us say that black represents liability), once both of the drawn balls are revealed, is 55% by De Morgan's rule. In a sense, the act of revealing the balls to be white or black affects the odds, because the balls must be

[15] This graph, with slight alteration, comes from Timothy J. Ross & W. Jerry Parkinson, *Fuzzy Set Theory, Fuzzy Logic, and Fuzzy Systems*, *in* FUZZY LOGIC AND PROBABILITY APPLICATIONS 29, 33 fig.2.3 (Timothy J. Ross et al. eds., 2002).

either white or black and only one has to be black. If one turns up black, this takes the pressure off the other's being black.[16]

Third, for modeling event occurrence, imagine a claim A for tort, where the defendant was actually 25% likely to have done the very bad acts alleged, and an independent claim B for contract, where the defendant was actually 40% likely to have done the very bad acts alleged. Assume that all available evidence would not change those numbers, which is what I mean by the word "actually." Is this third situation more like the first or the second situation? All our intuitions, honed by life-long exposure to traditional probability theory, point us to the second. But the real world of imprecision and other uncertainty makes the appropriate analogy the first situation.

IV. FUZZY LOGIC JUSTIFIED

Now let me flesh out my counterargument. I start by noting that the law always asks whether the plaintiff has proven a fact by satisfying the standard of proof—regardless of the *nature* of the factual finding, be it a finding on an imprecise matter or a finding regarding random uncertainty. To aggregate the findings, the law applies the MAX rule—it is thereby saying that a number of theories *almost* proved do not combine to produce liability.

There are many reasons for the law to use fuzzy logic in thinking about the real world. In the course of legal fact-finding, fuzzy logic alone can handle the frequent situations where the issues involve event imprecision. It alone can handle the situation where the law must combine a finding of event imprecision, as in the first hypothetical case, with a finding of event occurrence, as in the third case. It alone can smoothly handle the situation where the two claims are not independent. And of course we would never want to ask real factfinders to make the probability determinations that De Morgan's rule necessitates for calculation.

Yet I am not merely contending that fuzzy logic is convenient, but rather that it is the right vehicle for accurate fact-finding. As I have implied, the easy case for my position is the first of the three:

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[16] Clermont, *supra* note 4, at 19.

traditional probability cannot handle event imprecision (normative uncertainty) any better than it can handle Tom's tallness or smartness. But I am not conceding the harder third case of event occurrence (factual uncertainty). Thus, I contend the MAX rule is the way to go even when combining two independent and alternative findings on the likelihood that liability occurred in fact.

An initial step here is to observe that the law's embrace of the MAX rule seems commonsensical. A number of failures do not create a success. A plaintiff should not be able to raise the likelihood of success by framing alternative *independent* arguments.

Only by drawing inspiration from the betting table can scholars construct an argument to the contrary. Yet drawing balls from an urn is a bad analogy to fact-finding because we will never be able to lift the veil in order to see the color of the balls that represent facts found. In establishing facts in the courtroom, we will never know the full truth because all we can ever find is a degree of truth. Thus, we are not concerned with whether something is fully true—whether a truth value of 1 will turn up—but whether it seems to have a sufficient truth value to support liability. We are concerned not with whether we picked from the urn a black ball that we will never see, but rather with how close to black is this grayish ball that we can see.[17] De Morgan's rule thus answers a question that the law is not asking.

In the urn case, there is a 25% chance that the first urn's ball will turn up black. That 25% carries over to the second ball in the sense of increasing the odds to 55% that at least one of the balls will be black. Note, however, that if the ball from urn A is revealed to be white, the chance of the ball from urn B being black is 40%, not 55%.[18]

When the law finds tort claim A to be 25% likely, it is not saying that in twenty-five of a hundred cases liability will come up as 1. It is saying that a valid tort claim does not exist. In those

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[17] Or think of a somewhat sloppily drawn circle. Which is more appropriate for the law to ask: (1) whether there is a 90% probability that it is a perfect circle or (2) whether it has a .90 membership in the set of circles? *See* BART KOSKO, FUZZY THINKING: THE NEW SCIENCE OF FUZZY LOGIC 44–46 (1993) (providing this image).

[18] *See* Clermont, *supra* note 4, at 19 (explaining this point).

twenty-five cases, a finding of liability would be an error. Those twenty-five hypothetical decisions on claim A are not 1s to be distributed randomly among a hundred hypothetical decisions on contract claim B; they are errors to be discarded before going forward. When the law proceeds to claim B, no trace of the 25% liability on claim A carries over to affect the B inquiry. Claim B remains 40% likely.[19]

There lies the key to the paradox. The law has made a finding of partial truth on claim A, not a prediction on how things would turn out in a bivalent world. There is no sense in having fact-finders fix the odds of 1 turning up if only knowledge were perfect. The aim instead is to measure their belief in the facts based on imperfect evidence. Thus, even for a *single* fact subject only to random uncertainty of occurrence, the fact-finders' measure of likelihood is a belief rather than a traditional probability. Confusion arises because expression of a partial truth sounds so much like a traditional probability. But provability (the formation of a belief amidst uncertainty as to what happened that sufficiently justifies liability) is the law's concern, not probability (the odds that 1 will turn up).[20]

When fact-finders have to combine *multiple* facts' measures of likelihood, they should employ the operator for combining beliefs, not calculate the odds of 1s turning up. Belief that claim A failed and belief that claim B failed produce a belief that both claim A and claim B failed. That belief will be stronger than its negation, that is, the belief that one or the other claim, or both claims, succeeded. To minimize errors, the law should decide in accordance with that stronger belief. Consequently, the law should and does instruct the use of the mathematically sound way to combine beliefs, here the MAX rule.

Now reconsider the third case of a 25% probable tort claim and a 40% probable contract claim. More complete evidence will not

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[19] Indeed, if claim A's result bore on the independent claim B, the result would be relevant evidence to prove claim B. It is not, because logically claim A's result has nothing to say about how to decide claim B.

[20] *See* L. JONATHAN COHEN, THE PROBABLE AND THE PROVABLE 89–91, 265–67 (1977) (arguing that the law's interest in provability means that traditional probability's product rules should not apply, in view of the "inapplicability of betting odds" to "unsettlable issues").

arrive to change the likelihoods—and we cannot lift the veil to show what really happened. We will not get to see if the "ball" is truly black; we will never get to reduce the world to bivalence. It was the reduction to bivalence that affected the joint odds in the urn case. Unlike the drawn balls whose probabilities will change upon unveiling, the likelihoods in the third case will never be anything but 25% and 40%. If we can never convert the likelihood of a claim to one or zero, then all we can say is that the defendant is liable to a certain degree. That is, when we can never know with certainty what happened, a likelihood of occurrence is no different from a degree of misfeasance: a likelihood of occurrence is not a traditional probability, it is a fuzzy set.

Finally, consider the strongest argument in rebuttal. It would concede that fuzzy logic might apply in most cases, but it would maintain that traditional probability should apply in a pure hypothetical of event occurrence. So, the rebutter would have us imagine a fact-finder being 40% sure that a driver caused each of ten different car accidents, where "cause" is unrealistically assumed to have an accepted meaning and to be an on/off matter (note that the issue is causation rather than fault, and note that the fact of multiple independent accidents is not relevant to causation of any particular accident, which stays at 40% likely). We may address this case as follows:

(1) Does the fact-finder think, if he or she could only lift the veil, that causation of at least one of the accidents would have a truth value of 1? Yes, of course. In this betting situation, the residual chance of causation carries over, as risk, from one accident to the next. De Morgan's rule gives the risk that at least one accident will occur, but it does not tell us which one.

(2) Does the fact-finder believe that the driver caused any specific accident? No, if the issue is causation of an accident, not free-floating risk of a nonspecified accident. If the fact-finder believes, based on the evidence, that the driver did not cause the first accident or the second one or the

other ones, then the fact-finder will believe that the driver did not cause any particular accident. That is, the driver having caused any of the ten accidents is a .40 member of the set of true statements.

(3) Should we change tort law to impose liability on proof of negligent behavior that imposed a significant risk but without proof of its causing any particular accident in suit? Maybe, although that question is a complicated one of substantive law. There hides the trick in the rebuttal. The argument starts to answer how probability works in determining cause, but it switches the question to whether the law should impose liability for risk without causation. *That latter question is a matter of substantive policy, not a puzzle of probability.* Certainly, we should not adopt any such substantive reform on the basis of faulty arguments that probability theory dictates the reform.[21]

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[21] Professors Porat and Posner's cited exceptions, where the law might seem to aggregate by multiplying probabilities—for example, market-share liability, Porat & Posner, *supra* note 4, at 26–27—actually constitute changes to the substantive law rather than exceptional applications of traditional probability to fact-finding. In a manner comparable to the imposition of strict liability, market-share liability changes the elements of the cause of action (here substituting risk of harm for cause in fact) not at all for probability reasons but solely for substantive reasons. *Cf.* Mark A. Geistfeld, *The Doctrinal Unity of Alternative Liability and Market-Share Liability*, 155 U. PA. L. REV. 447, 471–77 (2006) (attempting to make market-share liability look less radical).

The authors also draw illustrations from the realm of proportional liability. Porat & Posner, *supra* note 4, at 6–7, 26. But whether a polluter caused a particular cancer is a factual question (on which one would not multiply probabilities) different from the legal question of whether tort law should impose liability for the pollution's increased risk of cancer (on which one could multiply probabilities). *See* David A. Fischer, *Proportional Liability: Statistical Evidence and the Probability Paradox*, 46 VAND. L. REV. 1201, 1201 (1993) (criticizing, on substantive grounds, proposals that called for "modifying traditional tort rules to permit plaintiffs to recover from a defendant who contributed to the risk of causing the plaintiff's harm without proving that the defendant actually caused the harm"). For a completely different example of theirs, constitutional law's choice to apply strict scrutiny to hybrid free-exercise claims does not even involve fact-finding. Porat & Posner, *supra* note 4, at 48–49.

The bottom line is this: *as an artifact of bivalence, the product rules do not apply to subjective probabilities for fact-finding*. This position is not anti-probabilist. To the contrary, my position is that we should conceive probability correctly. As to aggregation, the law is not asking for a measure of the probability that one of the claims will reveal itself to be completely true rather than completely false. Instead, the law seeks the likelihood of one of the claims being sufficiently true to impose liability.

I therefore submit that degrees of truth behave more appropriately here than bivalent probabilities, as the law has long recognized. That recognition constitutes a powerful piece of evidence—adding to many other pieces that I have marshaled elsewhere[22]—that the law's standard of proof employs fuzzy logic in preference to classical logic.

V. CONCLUSION

The aggregation paradox would arise only if theory really calls for application of De Morgan's rule. But theory does not. Instead, sound theory invokes the MAX rule, just as the law does. The truth of the aggregation equals the maximum of the findings' truths. Thus, there is no aggregation paradox. It implodes under the force of fuzzy logic.

The law seems to know what it is doing.

That there is no connection between their examples is unsurprising, as there is no stopping point to their theory of probabilistic aggregation, and so it straddles many settings. Yet they do take some reforms off the table. For example, unlike aggregation as to causation, they would not aggregate "almost negligent" behaviors, given that "two or more instances of 'almost negligent' behavior are even more socially desirable than one." *Id.* at 67. Why this limit to their aggregation? The reason is that in their view, the rejected reforms are undesirable as a matter of substantive law. *See id.* (referencing "cases where normative aggregation makes little sense because of the nature of the substantive law in question"). The rejected reforms are, however, just as much of an offense to their understanding of probability theory. Thus, the authors' arguments about probability theory are irrelevant to the line they draw between desirable and undesirable reforms.

[22] KEVIN M. CLERMONT, STANDARDS OF DECISION IN LAW: PSYCHOLOGICAL AND LOGICAL BASES FOR THE STANDARD OF PROOF, HERE AND ABROAD ch. V (forthcoming 2013); Clermont, *supra* note 4; Clermont, *supra* note 1.